

Comments on "Structural dynamics and resonance in plants with nonlinear stiffness"

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Abstract

We make comments on the paper by Miller [*J. Theor. Biol.* **234** (2005) 511].

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Recently researchers suggested that trees, crops, and other plants often uproot or snap when they are forced by gusting winds or waves at their natural frequency (see, e.g., Kerzenmacher and Gardiner, 1998). This can be attributed to the fact that the deflections of the plant, and hence mechanical stresses along the stem and root system, are greatest during resonance (cf. Spatz and Speck, 2002). Motivated by above status, Miller just reported the effects of hardening (elastic modulus increases with strain) and softening (elastic modulus decreases with strain) nonlinearities on the structural dynamics of plant stems (Miller, 2005). To better understand the effect of nonlinear stiffness on the resonant behavior of plants, Miller modeled plant stems as forced Duffing oscillators with softening or hardening nonlinearities and found that the resonant behavior of plants with nonlinear stiffness is in the large different from that predicted by linear models of plant structural dynamics. In the reported results, the maximum amplitudes of deflection of the plant stem were calculated numerically for forcing frequencies ranging from zero to twice the natural frequency. For hardening nonlinearities, the resonant behavior was 'pushed' to higher frequencies, and the maximum deflection amplitudes were lower than for the linear case. For softening nonlinearities, the resonant behavior was pushed to lower frequencies, and the maximum deflection amplitudes were higher than for the linear case. However, Miller found that damping has the effect of drastically decreasing deflection amplitudes and reducing the effect of the nonlinearities.

Some of the details could be introduced below before the present author's comments Miller modeled the plant as a system via a Duffing oscillator, a cubic stiffness term, $k_3 x^3(t)$; is added to the left-hand side of conventional linear ordinary differential equation (ODE) describing a spring-mass-damper system : $m x''(t) + c x'(t) + k_1 x(t) = F(t)$, where m is the mass, c is the damping coefficient, k_1 is proportional to the stiffness of the spring, and $F(t)$ is the applied force as a function of time. Here, k_3 is the cubic stiffness coefficient. $k_3 > 0$ models a hardening nonlinearity, and $k_3 < 0$ models a softening nonlinearity. $x(t)$ gives the displacement of the stem as a function of time. This displacement may be described as the horizontal displacement

or angular displacement of a point on the stem. The choice of $x(t)$ determines the values of the corresponding constants c and k_1 . In terms of plant structural dynamics, the first term of the equation describes the effective mass of the plant times its acceleration, the second term describes the friction in the system due to aerodynamic, material and structural damping of the plant, the third term describes the resistance of the trunk or stem to bending, and the forcing term describes the effective wind force on the plant. In reality, the damping term is probably not linearly proportional to velocity. Note also that this equation gives motion at one point.

Soft and hard nonlinear structures have often been modeled as Duffing oscillators (cf. the detailed references in Miller, 2005; the Duffing oscillator was originally introduced to model the large amplitude vibration modes of a steel beam subjected to periodic forces; Duffing, 1918). Since most biomaterials have nonlinear stiffness, modeling biological structures as Duffing oscillators could also be beneficial in gaining a better understanding of resonance in plants. Miller explained that the justification for this choice of model is based on history and simplicity: this is a well-studied nonlinear model that is relatively simple, lends itself to some analytical work, and yet has very complex behavior (Miller, 2005).

The addition of this nonlinear cubic term is what distinguishes this model from previously proposed models of plant stem dynamics. An argument can be made that the Duffing oscillator is the simplest appropriate nonlinear model available. A linear term and a nonlinear term are needed to describe the initial linear-elastic region of the stress-strain curve and the subsequent nonlinearities under large deformations.

The following change of variables was made (Miller, 2005): $X = x/L_{max}$, $T = \omega_o t$, where L_{max} is a characteristic maximum deflection, $\omega_o = \sqrt{k_1/m}$ is the natural frequency. In this case, L_{max} is the deflection at which failure occurs.

$$X''(T) + 2\xi X'(T) + X(T) + \frac{K_3}{K_1} X^3(T) = \frac{p_o}{K_1} F\left(\frac{\omega}{\omega_o} T\right), \quad (1)$$

here, $\xi = c/(2m\omega_o)$ (the damping ratio), $K_1 = k_1 L_{max}$, $K_3 = k_3 (L_{max})^3$, p_o is the forcing amplitude.

Miller's simulations were run for 40 periods of oscillation. For each set of parameters, 'forward' and 'backward' scans were performed. This technique was used to ensure that the upper portion of the response curve was simulated since the solution of the Duffing equation is dependent upon the initial conditions (Tufillaro et al., 1992). The forward and backward scans were performed as follows: during the first 10 oscillations, the forcing frequency was set to a low frequency ($\omega = 0.1\omega_o$) for the forward scan or a high frequency ($\omega = 2.0\omega_o$) for the backward scan. During the next 30 oscillations, the forcing frequency was set to the forcing frequency considered for the simulation. The maximum deflection amplitude was taken over the last 20 periods of oscillation. These maximum amplitudes were plotted as functions of the forcing frequency, damping ratio, and nonlinear ratio (Miller, 2005). For a certain range of forcing frequencies, the response amplitudes become multi-valued. This corresponds to forcing frequencies that yield more than one real root (cf. Eq. (11) in Miller, 2005). In the damped case, the response amplitudes become multi-valued if the damping is sufficiently small (cf. Fig. 4 in Miller, 2005). In the multi-valued range of the graph, the response amplitudes can take on any value of the branches. The value

taken depends upon the initial conditions.

The present author likes to make some remarks about Miller's interesting paper (Miller, 2005). Firstly, while Miller used an effective-mass approach, but the plant mass was treated as a fixed quantity which is not realistic for a plant system since the mass of a living plant is not fixed (there are gain and loss of its mass considering the photosynthesis and the water-evaporation of the leaves and the fluid flowing from the soil into the root of the plant). As the wind passes by, once there are stresses or strains to the plant, the water content inside the plant will subsequently change and thus the mass of the plant is changing (cf. Chu, 2004)!

Secondly, the plant system is essentially a continuous (dynamical) system which has an infinite degrees of freedom. As Miller also noticed, asymmetries in the elastic properties or mass distribution along the stem could contribute to twisting motions and torsional strains. Thus, coupled twisting and bending could be significant to the resonant behavior of the plant. Even it was treated as one degree of freedom, the flow-structure interaction is still complicated (the plant (structure) disturbs the flow or wind and there is also a feedback from the flow (say, wind) to the (elastic) structure simultaneously) enough and it needs to be considered (the inertia, the damping and the stiffness could be altered via this flow-structure interaction). Meanwhile, the approximation of $x(t)$ (cf. Fig. 1 in Miller, 2005) is not valid once there is larger wind-loading to the plant : the root (network) of the plant will shift its position due to adaptive accommodations which then changes $x(t)$. In fact, the fixed-end support for the plant could be relaxed to a support which can slip along the boundary.

Thirdly, although Miller adopted a nonlinear approach for a simple (one degree of freedom) system. What is the benefit of the Miller's nonlinear approach compared to the linear approach for several degrees of freedom or continuous (elastic) system model? As we know, the (non-linear) anharmonic term in damped Duffing equation (cubic term : $k_3[x(t)]^3$) is nothing but to induce a phase shift (Duffing, 1918; Denman, 1998; Vega and Knobloch, 2003) which Miller could numerically obtain and present them in Figures 7 and 9 (cf. Miller, 2005) for hardening and softening nonlinearities.

Fourthly, as Miller noted : *Plant structures have a unique capability of regularly withstanding substantial resonant loading. Similar to the hardening case, a 'soft' plant might also avoid the effects of resonance by moving resonance away from the driving frequency, if the driving frequency is relatively constant.* But, Miller could only remark that *to gain a complete understanding of how plants deal with resonance and why they fail, further work describing the behavior of plant stems beyond their linear elastic range is needed. With the use of detailed data on how the elastic modulus varies with deformation, more sophisticated models of plant stem dynamics could be developed. The ODE model of the Duffing oscillator could also be extended to a continuous partial differential equation model. Such a model could describe deflection, mass, and stiffness as a function of height.* (the last statement is relevant to the second remark the present author made above) Miller didn't provide the detailed clues how to tune or accommodate the natural noises. It seems to the present author that the plant subjected to dynamic wind-loading could be a multistable oscillator (cf. Blekhman and Landa, 2004; Mikhlin and Manucharyan, 2003) as there might be several equilibrium positions. For example, using a bistable oscillator described

by a Duffing equation as an example, resonances caused by a biharmonic external force with two different frequencies (the so-called vibrational resonances) were considered (Blekhman and Landa, 2004). It was shown that, in the case of a weakly damped oscillator, these resonances are conjugate; they occur as either the low and high frequency is varied. In addition, the resonances occur as the amplitude of the high-frequency excitation is varied. It was also shown that the high-frequency action induces the change in the number of stable steady states; these bifurcations are also conjugate, and are the cause of the seeming resonance in an overdamped oscillator.

Finally, it seems the multi-scale (either to the time or to the 1-dimensional space) approach is useful to the present problem as the spatial-temporal response for the leaves, the stem, and the root (network) of the entire plant would be rather different (e.g., just to consider the deflection or strain for the leaves, the stem, and the root of the plant subjected to the wind-loading!).

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